DAY THIRTY

Unit Test 4
(Coordinate Geometry)

1 The parabola $y^2 = 4x$ and the circle $(x-6)^2 + y^2 = r^2$ will have no common tangent, then

(a) $r > \sqrt{20}$ (b) $r < \sqrt{20}$
(c) $r > \sqrt{18}$ (d) $r \in (\sqrt{20}, \sqrt{28})$

2 The lines $lx + my + n = 0$, $mx + ny + l = 0$ and $nx + ly + m = 0$ are concurrent, if

(a) $l + m + n = 0$ (b) $l + m - n = 0$
(c) $l - m + n = 0$ (d) None of these

3 If the latusrectum of a hyperbola through one focus substends 60° angle at the other focus, then its eccentricity e is

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{5}$ (d) $\sqrt{6}$

4 Set of values of m for which a chord of slope m of the circle $x^2 + y^2 = 4$ touches the parabola $y^2 = 4x$, is

(a) $\left(-\infty, -\sqrt{\frac{\sqrt{2}-1}{2}}\right) \cup \left(\sqrt{\frac{\sqrt{2}-1}{2}}, \infty\right)$
(b) $(-\infty, 1) \cup (1, \infty)$
(c) $(-1, 1)$
(d) R

5 If ω is one of the angles between the normals to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the points whose eccentric angles are θ and $\frac{\pi}{2} + \theta$, then $\frac{2\cot\omega}{\sin 2\theta}$ is equal to

(a) $\frac{e^2}{\sqrt{1-e^2}}$ (b) $\frac{e^2}{\sqrt{1+e^2}}$ (c) $\frac{e^2}{1-e^2}$ (d) $\frac{e^2}{1+e^2}$

6 If in a ΔABC (whose circumcentre is origin), $a \le \sin A$, then for any point (x, y) inside the circumcircle of ΔABC

(a) $|xy| < \frac{1}{8}$ (b) $|xy| > \frac{1}{8}$
(c) $\frac{1}{8} < xy < \frac{1}{2}$ (d) None of these

7 If $A(n, n^2)$ (where, $n \in N$) is any point in the interior of the quadrilateral formed by the lines $x = 0$, $y = 0$, $3x + y - 4 = 0$ and $4x + y - 21 = 0$, then the possible number of positions of the point A is

(a) 0 (b) 1
(c) 2 (d) 3

8 The range of values of r for which the point $\left(-5 + \frac{r}{\sqrt{2}}, -3 + \frac{r}{\sqrt{2}}\right)$ is an interior point of the major segment of the circle $x^2 + y^2 = 16$, cut off by the line $x + y = 2$, is

(a) $(-\infty, 5\sqrt{2})$
(b) $(4\sqrt{2} - \sqrt{14}, 5\sqrt{2})$
(c) $(4\sqrt{2} - \sqrt{14}, 4\sqrt{2} + \sqrt{14})$
(d) None of the above

9 AB is a double ordinate of the parabola $y^2 = 4ax$. Tangents drawn to parabola at A and B meet Y-axis at A_1 and B_1, respectively. If the area of trapezium AA_1B_1B is equal to $24a^2$, then angle subtended by A_1B_1 at the focus of the parabola is equal to

(a) $2\tan^{-1}(3)$ (b) $\tan^{-1}(3)$
(c) $2\tan^{-1}(2)$ (d) $\tan^{-1}(2)$

10 If two tangents can be drawn to the different branches of hyperbola $\frac{x^2}{1} - \frac{y^2}{4} = 1$ from the point (α, α^2), then

(a) $\alpha \in (-2, 0)$ (b) $\alpha \in (-3, 0)$
(c) $\alpha \in (-\infty, -2)$ (d) $\alpha \in (-\infty, -3)$

11 A hyperbola has the asymptotes $x + 2y = 3$ and $x - y = 0$ and passes through $(2, 1)$. Its centre is

(a) (1, 2) (b) (2, 2)
(c) (1, 1) (d) (2, 1)

12 The equation of the ellipse having vertices at $(\pm 5, 0)$ and foci at $(\pm 4, 0)$ is

(a) $\frac{x^2}{25}+\frac{y^2}{16}=1$ (b) $4x^2+5y^2=20$

(c) $9x^2+25y^2=225$ (d) None of these

13 The foci of an ellipse are $(0, \pm 1)$ and minor axis is of unit length. The equation of the ellipse is

(a) $2x^2+y^2=2$ (b) $x^2+2y^2=2$

(c) $4x^2+20y^2=5$ (d) $20x^2+4y^2=5$

14 The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and having its centre at (0, 3) is

(a) 3 (b) 4

(c) $\sqrt{12}$ (d) $\frac{7}{2}$

15 If the straight lines $2x+3y-1=0$, $x+2y-1=0$ and $ax+by-1=0$ form a triangle with origin as orthocentre, then (a, b) is given by

(a) $(-3, 3)$ (b) $(6, 4)$

(c) $(-8, 8)$ (d) $(0, 7)$

16 If $P(1, 0)$, $Q(-1, 0)$ and $R(2, 0)$ are three given points, then the locus of S satisfying the relation $SQ^2+SR^2=2SP^2$ is

(a) a straight line parallel to X-axis

(b) a circle through the origin

(c) a circle with centre at the origin

(d) a straight line parallel to Y-axis

17 The point $(a^2, a+1)$ lies in the angle between the lines $3x-y+1=0$ and $x+2y-5=0$ containing the origin, if

(a) $a \in (-3, 0) \cup \left(\frac{1}{3}, 1\right)$ (b) $a \in (-\infty, -3) \cup \left(\frac{1}{3}, 1\right)$

(c) $a \in \left(-3, \frac{1}{3}\right)$ (d) $a \in \left(\frac{1}{3}, \infty\right)$

18 The diameter of $16x^2-9y^2=144$ which is conjugate to $x=2y$ is

(a) $y=\frac{16x}{9}$ (b) $y=\frac{32x}{9}$

(c) $x=\frac{16y}{9}$ (d) $x=\frac{32y}{9}$

19 The locus of poles with respect to the parabola $y^2=12x$ of tangent to the hyperbola $x^2-y^2=9$ is

(a) $4x^2+y^2=36$ (b) $x^2+4y^2=9$

(c) $x^2+4y^2=36$ (d) $4x^2+y^2=81$

20 A point moves such that the area of the triangle formed by it with the points (1, 5) and (3, – 7) is 21 sq units. Then, locus of the point is

(a) $6x+y-32=0$ (b) $6x-y+32=0$

(c) $6x-y-32=0$ (d) $x+6y-32=0$

21 The length of the latusrectum of the parabola $169[(x-1)^2+(y-3)^2]=(5x-12y+17)^2$ is

(a) $\frac{14}{13}$ (b) $\frac{28}{13}$ (c) $\frac{12}{13}$ (d) $\frac{16}{13}$

22 If two vertices of an equilateral triangle are (0, 0) and $(3, 3\sqrt{3})$, then the third vertex is

(a) (3, – 3) (b) (– 3, 3)

(c) $(-3, 3\sqrt{3})$ (d) None of these

23 Let ABC is a triangle with vertices $A(-1, 4)$, $B(6, -2)$ and $C(-2, 4)$. If D, E and F are the points which divide each AB, BC and CA respectively, in the ratio 3 : 1 internally. Then, the centroid of the ΔDEF is

(a) (3, 6) (b) (1, 2)

(c) (4, 8) (d) None of these

24 A variable circle through the fixed point $A(p, q)$ touches the X-axis. The locus of the outer end of the diameter through A is

(a) $(x-p)^2=4qy$ (b) $(x-q)^2=4py$

(c) $(x-p)^2=4qx$ (d) $(x-q)^2=4px$

25 The exhaustive range of values of a such that the angle between the pair of tangents drawn from (a, a) to the circle $x^2+y^2-2x-2y-6=0$ lies in the range $\left(\frac{\pi}{3}, \pi\right)$ is

(a) $(0, \infty)$ (b) $(-3, -1) \cup (3, 5)$

(c) $(-2, -1) \cup (2, 3)$ (d) $-(3, 0) \cup (1, 2)$

26 The four distinct points (0, 0), (2, 0), (0, – 2) and $(k, -2)$ are concyclic, if k is equal to

(a) 0 (b) – 2 (c) 2 (d) 1

27 If a point $P(4, 3)$ is rotated through an angle 45° in anti-clockwise direction about origin, then coordinates of P in new position are

(a) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (b) $\left(-\frac{7}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

(c) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (d) $\left(\frac{1}{\sqrt{2}}, -\frac{7}{\sqrt{2}}\right)$

28 The number of integral values of λ for which the equation $x^2+y^2-2\lambda x+2\lambda y+14=0$ represents a circle whose radius cannot exceed 6, is

(a) 9 (b) 10 (c) 11 (d) 12

29 The slopes of tangents to the circle $(x-6)^2+y^2=2$ which passes through the focus of the parabola $y^2=16x$ are

(a) ± 2 (b) 1/2, –2

(c) –1/2, 2 (d) ± 1

30 The range of values of n for which $(n, -1)$ is exterior to both the parabolas $y^2=|x|$ is

(a) (0, 1) (b) (– 1, 1)

(c) (– 1, 0) (d) None of these

31 The parameters t and t' of two points on the parabola $y^2=4ax$, are connected by the relation $t=k^2t'$. The tangents at these points intersect on the curve

(a) $y^2=ax$ (b) $y^2=k^2x$

(c) $y^2=ax\left(k+\frac{1}{k}\right)^2$ (d) None of these

32 Triangle ABC is right angled at A. The circle with centre A and radius AB cuts BC and AC internally at D and E respettvely if $BD = 20$ and $DC = 16$ then the length of AC equals.

(a) $6\sqrt{21}$ (b) $6\sqrt{26}$ (c) 30 (d) 32

33 If the line $y - \sqrt{3}x + 3 = 0$ cuts the parabola $y^2 = x + 2$ at A and B, and if $P \equiv (\sqrt{3}, 0)$, then $PA \cdot PB$ is equal to

(a) $2\left(\frac{2+\sqrt{3}}{1}\right)$ (b) $\frac{4(2-\sqrt{3})}{3}$

(c) $\frac{4(2+\sqrt{3})}{3}$ (d) $\frac{2(2-\sqrt{3})}{3}$

34 If S and S' are the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and P is any point on it, then difference of maximum and minimum of $SP \cdot S'P$ is equal to

(a) 16 (b) 9 (c) 15 (d) 25

35 The locus of a point which moves, such that the chord of contact of the tangent from the point to two fixed given circles are perpendicular to each other is

(a) circle (b) parabola

(c) ellipse (d) None of these

36 Tangent is drawn to the ellipse $\frac{x^2}{27} + y^2 = 1$ at $(3\sqrt{3}\cos\theta, \sin\theta)\left[\text{where}, \theta \in \left(0, \frac{\pi}{2}\right)\right]$. Then, the value of θ such that sum of intercept on axes made by this tangent is minimum, is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{8}$ (d) $\frac{\pi}{4}$

37 The condition for the line $px + qy + r = 0$ to be tangent to the rectangular hyperbola $x = ct$, $y = \frac{c}{t}$ is

(a) $p < 0, q > 0$ (b) $p > 0, q > 0$

(c) $p > 0, q < 0$ (d) None of these

38 If the line $x + 3y + 2 = 0$ and its perpendicular line are conjugate w.r.t. $3x^2 - 5y^2 = 15$, then equation of conjugate line is

(a) $3x - y = 15$ (b) $3x - y + 12 = 0$

(c) $3x - y + 10 = 0$ (d) $3x - y = 4$

39 The product of the lengths of perpendicular drawn from any point on the hyperbola $x^2 - 2y^2 - 2 = 0$ to its asymptotes, is

(a) 1/2 (b) 2/3 (c) 3/2 (d) 2

40 Tangents at any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ cut the axes at A and B respectively, if the rectangle $OAPB$, where O is the origin is completed, then locus of the point P is given by

(a) $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$ (b) $\frac{a^2}{k^2} + \frac{b^2}{y^2} = 1$

(c) $\frac{a^2}{y^2} - \frac{b^2}{x^2} = 1$ (d) None of these

41 In a triangle ABC, if $A(2, -1)$ and $7x - 10y + 1 = 0$ and $3x - 2y + 5 = 0$ are equation of on altitude and on angle bisector respectively drawn from B, the equation of BC is.

(a) $x + y + 1 = 0$

(b) $5x + y + 17 = 0$

(c) $4x + 9y + 30 = 0$

(d) $x - 5y - 7 = 0$

42 If $\frac{a}{\sqrt{bc}} - 2 = \sqrt{\frac{b}{c}} + \sqrt{\frac{c}{b}}$ where $a, b, c > 0$ the family of lines $\sqrt{a}x + \sqrt{b}y + \sqrt{c} = 0$ passes through the point.

(a) (1, 1) (b) (1, – 2) (c) (–1, 2) (d) (– 1, 1)

43 A series of ellipses $E_1, E_2, E_3, \ldots, E_n$ are drawn such that E_n touches E_{n-1} at the extremeties of the major axis of E_{n-1} and the foci of E_n coincide with the extremeties of minor axis of E_{n-1}. If eccentricity of the ellipse B independent of x then the value of eccentricity is.

(a) $\frac{\sqrt{5}-1}{2}$ (b) $\frac{\sqrt{5}+1}{3}$

(c) $\frac{\sqrt{5}+1}{4}$ (d) $\frac{\sqrt{5}-1}{4}$

44 If one of the diagonal of a square is along the line $x = 2y$ and one of its vertices is (3, 0), then its sides through this vertex are given by the equations

(a) $y - 3x + 9 = 0, 3y + x - 3 = 0$

(b) $y + 3x + 9 = 0, 3y + x - 3 = 0$

(c) $y - 3x + 9 = 0, 3y - x + 3 = 0$

(d) $y - 3x + 3 = 0, 3y + x + 9 = 0$

45 Given $A(0,0)$ and $B(x, y)$ with $x \in (0,1)$ and $y > 0$. Let the slope of line AB equals m_1. Point C lies on the line $x = 1$ such that the slope of BC equals m_2 where $0 < m_2 < m_1$. If the area of the triangle ABC can be expressed as $(m_1 - m_2)f(x)$, then the largest possible value of $f(x)$ is.

(a) 1/8 (b) 1/2 (c) 1/4 (d) 1

46 A circle is inscribed into a rhombus $ABCD$ with one angle 60°. The distance from the centre of the circle to the nearest vertex is equal to 1. If P is any point of the circle, then

$|PA|^2 + |PB|^2 + |PC|^2 + |PD|^2$ is equal to.

(a) 8 (b) 9 (c) 10 (d) 11

47 The equation of common tangent touching the circle $(x-3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$ above the X-axis is

(a) $\sqrt{3}y = 3x + 1$ (b) $\sqrt{3}y = -(x + 3)$

(c) $\sqrt{3}y = x + 3$ (d) $\sqrt{3}y = -(3x + 1)$

48 If the tangent at the point ϕ on the ellipse $\frac{x^2}{16} + \frac{11y^2}{256} = 1$ touches the circle $x^2 + y^2 - 2x - 15 = 0$, then ϕ is equal to

(a) $\pm\frac{\pi}{2}$ (b) $\pm\frac{\pi}{4}$

(c) $\pm\frac{\pi}{3}$ (d) $\pm\frac{\pi}{6}$

Directions (Q.Nos. 49-55) *Each of these questions contains two statements : Statement* I *(Assertion) and Statement* II *(Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.*

(a) Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I

(b) Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I

(c) Statement I is true; Statement II is false

(d) Statement I is false; Statement II is true

49 If p, x_1, x_2, x_3 and q, y_1, y_2, y_3 form two arithmetic progression with common differences a and b.

Statement I The centroid of triangle formed by points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3), lies on a straight line.

Statement II The point (h, k) given by $h = \frac{x_1 + x_2 + \dots + x_n}{n}$ and $k = \frac{y_1 + y_2 + \dots + y_n}{n}$ always lies on the line $b(x - p) = a(y - q)$ for all values of n.

50 **Statement I** A is a point on the parabola $y^2 = 4ax$. The normal at A cuts the parabola again at point B.

If AB subtends a right angle at the vertex of the parabola, then slope of AB is $\frac{1}{\sqrt{2}}$.

Statement II If normal at $(at_1^2, 2at_1)$ cuts again the parabola at $(at_2^2, 2at_2)$, then $t_2 = -t_1 - \frac{2}{t_1}$.

51 Suppose $ABCD$ is a cyclic quadrilateral inscribed in a circle.

Statement I If radius is one unit and $AB \cdot BC \cdot CD \cdot DA \geq 4$, then $ABCD$ is a square.

Statement II A cyclic quadrilateral is a square, if its diagonals are the diameters of the circle.

52 If a circle $S = 0$ intersects a hyperbola $xy = c^2$ at four points.

Statement I If $c = 2$ and three of the intersection points are (2, 2), (4, 1) and $\left(6, \frac{2}{3}\right)$, then coordinates of the fourth point are $\left(\frac{1}{4}, 16\right)$.

Statement II If a circle intersects a hyperbola at t_1, t_2, t_3, t_4, then $t_1 \cdot t_2 \cdot t_3 \cdot t_4 = 1$.

53 The auxiliary circle of an ellipse is described on the major axis of an ellipse.

Statement I The circle $x^2 + y^2 = 4$ is auxiliary circle of an ellipse $\frac{x^2}{4} + \frac{y^2}{b^2} = 1$ (where, $b < 2$).

Statement II A given circle is auxiliary circle of exactly one ellipse.

54 The tangent at a point P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, which is not an extremity of major axis, meets a directrix at T.

Statement I The circle on PT as diameter passes through the focus of the ellipse corresponding to the directrix on which T lies.

Statement II PT subtends a right angle at the focus of the ellipse corresponding to the directrix on which T lies.

55 **Statement I** If the perpendicular bisector of the line segment joining $P(1, 4)$ and $Q(k, 3)$ has y-intercept -4, then $k^2 - 16 = 0$.

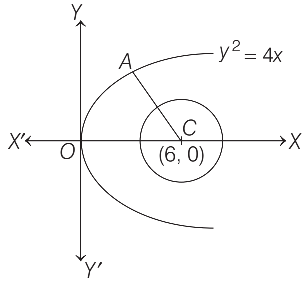
Statement II Locus of a point equidistant from two given points is the perpendicular bisector of the line joining the given points.

ANSWERS

1 (b)	**2** (a)	**3** (b)	**4** (a)	**5** (a)	**6** (a)	**7** (b)	**8** (b)	**9** (d)	**10** (c)
11 (c)	**12** (c)	**13** (d)	**14** (b)	**15** (c)	**16** (d)	**17** (a)	**18** (b)	**19** (a)	**20** (a)
21 (b)	**22** (c)	**23** (b)	**24** (a)	**25** (b)	**26** (c)	**27** (a)	**28** (c)	**29** (d)	**30** (b)
31 (c)	**32** (b)	**33** (c)	**34** (b)	**35** (a)	**36** (b)	**37** (b)	**38** (b)	**39** (b)	**40** (a)
41 (b)	**42** (d)	**43** (a)	**44** (a)	**45** (a)	**46** (d)	**47** (c)	**48** (c)	**49** (a)	**50** (d)
51 (c)	**52** (d)	**53** (c)	**54** (a)	**55** (a)					

Hints and Explanations

1 Any normal of parabola is
$y = -tx + 2t + t^3$.



If it passes through (6,0), then

$$-6t + 2t + t^3 = 0$$

$$\Rightarrow \quad t = 0, t^2 = 4$$

Thus, $\quad A \equiv (4, 4)$

Thus, for no common tangent,

$$AC = \sqrt{4 + 16} > r$$

$$\Rightarrow \quad r < \sqrt{20}$$

2 Since, lines are concurrent.

$$\therefore 1(lx + my + n) + 1(mx + ly + l) + 1(nx + ly + m) = 0$$

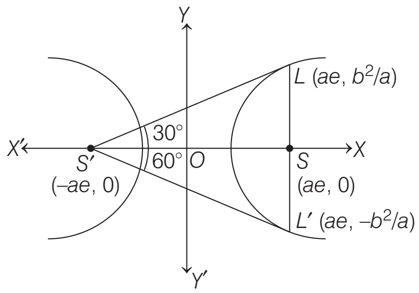
$$\Rightarrow x(l + m + n) + y(l + m + n) + (l + m + n) = 0$$

$$\therefore \quad l + m + n = 0$$

3 Let LSL' be a latusrectum through the focus $S(ae, 0)$ of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

It substends angle 60° at the otherfocus $S'(-ae, 0)$.



We have, $\angle LS'L' = 60°$

$$\therefore \quad \angle LS'S = 30°$$

In $\Delta LS'S$, $\tan 30° = \dfrac{LS}{S'S}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{b^2/a}{2ae}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{b^2}{2a^2e}$$

$$= \frac{e^2 - 1}{2e}$$

$$\Rightarrow \quad \sqrt{3}e^2 - 2e - \sqrt{3} = 0$$

$$\Rightarrow \quad (e - \sqrt{3})(\sqrt{3}e + 1) = 0$$

$$\therefore \quad e = \sqrt{3}$$

4 The equation of tangent of slope m to the parabola $y^2 = 4x$ is $y = mx + \dfrac{1}{m}$.

This will be a chord of the circle $x^2 + y^2 = 4$, if length of the perpendicular from the centre (0, 0) is less than the radius.

i.e. $\left|\dfrac{1}{m\sqrt{m^2 + 1}}\right| < 2$

$$\Rightarrow 4m^4 + 4m^2 - 1 > 0$$

$$\Rightarrow \left(m^2 - \frac{\sqrt{2} - 1}{2}\right)\left(m^2 + \frac{1 + \sqrt{2}}{2}\right) > 0$$

$$\Rightarrow \left(m^2 - \frac{\sqrt{2} - 1}{2}\right) > 0$$

$$\Rightarrow \left(m - \sqrt{\frac{\sqrt{2} - 1}{2}}\right)\left(m + \sqrt{\frac{\sqrt{2} - 1}{2}}\right) > 0$$

$$\Rightarrow m \in \left(-\infty, -\sqrt{\frac{\sqrt{2} - 1}{2}}\right) \cup \left(\sqrt{\frac{\sqrt{2} - 1}{2}}, \infty\right)$$

5 The equations of the normals to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ at the points whose eccentric angles are θ and $\dfrac{\pi}{2} + \theta$ are

$ax \sec\theta - by \operatorname{cosec}\theta = a^2 - b^2$

and $-ax \operatorname{cosec}\theta - by \sec\theta = a^2 - b^2$, respectively.

Since, ω is the angle between these two normals.

Therefore,

$$\tan\omega = \left|\frac{\frac{a}{b}\tan\theta + \frac{a}{b}\cot\theta}{1 - \frac{a^2}{b^2}}\right|$$

$$= \left|\frac{ab(\tan\theta + \cot\theta)}{b^2 - a^2}\right|$$

$$\Rightarrow \tan\omega = \left|\frac{2ab}{\sin 2\theta (b^2 - a^2)}\right|$$

$$= \frac{2ab}{(a^2 - b^2)\sin 2\theta} = \frac{2a^2\sqrt{1 - e^2}}{a^2e^2 \sin 2\theta}$$

$$\therefore \frac{2\cot\omega}{\sin 2\theta} = \frac{e^2}{\sqrt{1 - e^2}}$$

6 Given, $a \le \sin A \Rightarrow \dfrac{a}{\sin A} \le 1$

$$\Rightarrow 2R \le 1 \Rightarrow R \le \frac{1}{2}$$

So, for any point (x, y) inside the circumcircle,

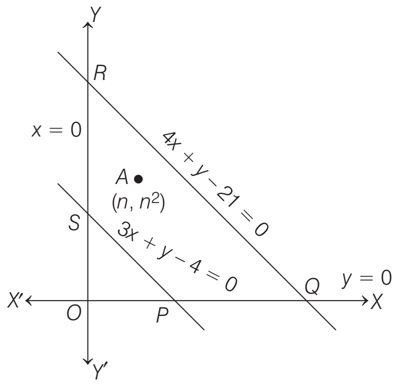
$$x^2 + y^2 < \frac{1}{4}$$

Using AM≥ GM, $\left(\dfrac{x^2 + y^2}{2} \ge |xy|\right)$

$$\Rightarrow \quad |xy| < \frac{1}{8}$$

7 Origin is on the left of PS, as

$$0 + 0 - 4 < 0$$



$\therefore$ At point $A(n, n^2)$,

$$3n + n^2 - 4 > 0$$

$$\Rightarrow \quad n^2 + 3n - 4 > 0$$

$$\Rightarrow \quad (n + 4)(n - 1) > 0$$

$$\Rightarrow \quad n - 1 > 0 \quad \ldots\text{(i)}$$

Now, as A and O lies on the same sides of QR.

and $\quad 4x + y - 21 = 0 + 0 - 21 < 0$

$\therefore$ At point $A(n, n^2)$,

$$4n + n^2 - 21 < 0$$

$$\Rightarrow \quad n^2 + 4n - 21 < 0$$

$$\Rightarrow \quad (n + 7)(n - 3) < 0$$

$$\Rightarrow \quad 0 < n < 3 \quad [\because n \in N] \ldots\text{(ii)}$$

From Eqs. (i) and (ii),

$$1 < n < 3 \Rightarrow n = 2$$

Hence, $A(2, 4)$ is only one point.

8 The given point is an interior point, if

$$\left(-5 + \frac{r}{\sqrt{2}}\right)^2 + \left(-3 + \frac{r}{\sqrt{2}}\right)^2 - 16 < 0$$

$$\Rightarrow \quad r^2 - 8\sqrt{2}r + 18 < 0$$

$$\Rightarrow 4\sqrt{2} - \sqrt{14} < r < 4\sqrt{2} + \sqrt{14}$$

Since, the point is on the major segment, the centre and the point are on the same side of the line $x + y = 2$.

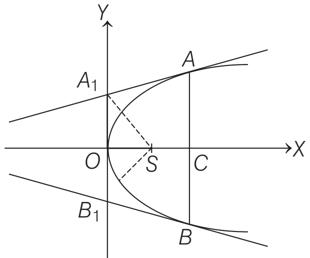
⇒ $-5+\frac{r}{\sqrt{2}}-3+\frac{r}{\sqrt{2}}-2<0$

⇒ $r<5\sqrt{2}$

Hence,

$4\sqrt{2}-\sqrt{14}<r<5\sqrt{2}$

9 Let $A\equiv(at_1^2, 2at_1)$, $B\equiv(at_1^2,-2at_1)$.
Equation of tangents at A and B are $yt_1 = x+at_1^2$ and $yt_1 = x-at_1^2$, respectively.
Now, $A_1\equiv(0, at_1)$, $B_1=(0,-at_1)$
Area of trapezium
$AA_1B_1B=\frac{1}{2}(AB+A_1B_1)\cdot OC$



⇒ $24a^2=\frac{1}{2}\cdot(4at_1+2at_1)(at_1^2)$

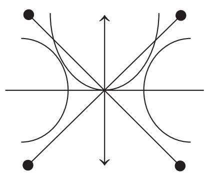
⇒ $t_1^3=8 \Rightarrow t_1=2 \Rightarrow A_1=(0,2a)$

If $\angle OSA_1=\theta$, then

$\tan\theta=\frac{2a}{a}=2 \Rightarrow \theta=\tan^{-1}(2)$

10 Given that, $\frac{x^2}{1}-\frac{y^2}{4}=1$

Since, (α,α^2) lies on the parabola $y=x^2$, then (α,α^2) must lie between the asymptotes of hyperbola $\frac{x^2}{1}-\frac{y^2}{4}=1$ in 1st and 2nd quadrant.



So, the asymptotes are $y=\pm 2x$.

∴ $2\alpha<\alpha^2 \Rightarrow \alpha<0$ or $\alpha>2$

and $-2\alpha<\alpha^2$

$\alpha<-2$ or $\alpha>0$

∴ $\alpha\in(-\infty,-2)$ or $(2,\infty)$

11 Given equations of asymptotes are

$x+2y=3$ ...(i)

and $x-y=0$ ...(ii)

On solving Eqs. (i) and (ii), we get

$x=1, y=1$

So, the centre of hyperbola is (1, 1).

12 The line joining foci and vertices is X-axis and the centre is (0, 0). So, axes of the ellipse coincide with coordinate axes.

Here, $a=5$ and $ae=4$

⇒ $e=\frac{4}{5}$

Now, $b^2=a^2(1-e^2)$

$=25\left[1-\left(\frac{4}{5}\right)^2\right]=9$

Hence, the equation of the ellipse is

$\frac{x^2}{5^2}+\frac{y^2}{3^2}=1$

⇒ $9x^2+25y^2=225$

13 Given, $2b=1 \Rightarrow b=\frac{1}{2}$ and $a\cdot e=1$

Since, $a^2(1-e^2)=\frac{1}{4}$

⇒ $a^2-1^2=\frac{1}{4} \Rightarrow a^2=\frac{5}{4}$

Hence, the equation of the ellipse

$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ is $\frac{x^2}{1/4}+\frac{y^2}{5/4}=1$ or

$20x^2+4y^2=5$.

14 Since, $a^2(1-e^2)=9$

⇒ $16-a^2e^2=9 \Rightarrow ae=\sqrt{7}$

So, foci are at $(\sqrt{7},0)$ and $(-\sqrt{7},0)$.

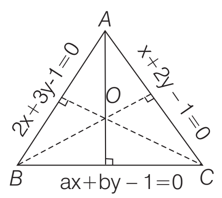
∴ Required radius

$=\sqrt{(\sqrt{7}-0)^2+(3-0)^2}=4$

15 Equation of AO is

$2x+3y-1+\lambda(x+2y-1)=0$.

Since, it passes through (0, 0), then $\lambda=-1$.



∴ $x+y=0$

Since, AO is perpendicular to BC.

∴ $(-1)\left(-\frac{a}{b}\right)=-1 \Rightarrow a=-b$

Similarly,

$(2x+3y-1)+\mu(ax-ay-1)=0$

will be equation of BO for $\mu=-1$.

Thus, BO is perpendicular to AC.

⇒ $-\frac{2-a}{3+a}\cdot\left(\frac{-1}{2}\right)=-1$

⇒ $2-a=-6-2a$

⇒ $a=-8$ and $b=8$

16 Let the coordinates of a point S be (x, y).

Since, $SQ^2+SR^2=2SP^2$

⇒ $(x+1)^2+y^2+(x-2)^2+y^2$
$=2[(x-1)^2+y^2]$

⇒ $2x+3=0$

Hence, it is a straight line parallel to Y-axis.

17 Since, origin and the point $(a^2, a+1)$ lie on the same side of both the lines.

∴ $3a^2-(a+1)+1>0$

and $a^2+2(a+1)-5<0$

i.e. $a(3a-1)>0$ and $a^2+2a-3<0$

i.e. $a(3a-1)>0$ and $(a-1)(a+3)<0$

⇒ $a\in(-\infty,0)\cup\left(\frac{1}{3},\infty\right)$

and $a\in(-3,1)$

∴ $a\in(-3,0)\cup\left(\frac{1}{3},1\right)$

18 Diameters $y=m_1x$ and $y=m_2x$ are conjugate diameters of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, if $m_1m_2=\frac{b^2}{a^2}$.

Here, $a^2=9$, $b^2=16$ and $m_1=\frac{1}{2}$

∵ $m_1m_2=\frac{b^2}{a^2}$

⇒ $\frac{1}{2}(m_2)=\frac{16}{9}$

⇒ $m_2=\frac{32}{9}$

Thus, the required diameter is $y=\frac{32x}{9}$.

19 Let the pole be (h,k), so that polar is

$ky=6(x+h)$

⇒ $y=\frac{6x}{k}+\frac{6h}{k}$

Since, it is tangent to the hyperbola,

$x^2-y^2=9$

∴ $c^2=9m^2-9$

⇒ $\frac{36h^2}{k^2}=\frac{324}{k^2}-9$

$\left[\because c=\frac{6h}{k}, m=\frac{6}{k}\right]$

⇒ $4h^2+k^2=36$

Hence, the locus is $4x^2+y^2=36$.

20 Let (x, y) be the required point.

Then, $\frac{1}{2}\begin{vmatrix} x & y & 1 \\ 1 & 5 & 1 \\ 3 & -7 & 1 \end{vmatrix}=21$

⇒ $(5+7)x-(1-3)y+(-7-15)=42$

⇒ $12x+2y-22=42$

⇒ $6x+y-32=0$

21 Given parabola is

$(x-1)^2+(y-3)^2=\left(\frac{5x-12y+17}{13}\right)^2$

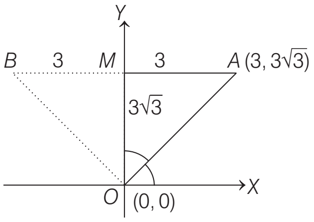
Focus = (1, 3), directrix is

$5x-12y+17=0$

∴ Length of latusrectum

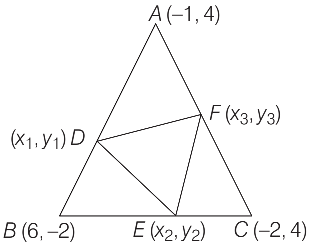
$=2\left|\frac{5-36+17}{13}\right|=\frac{28}{13}$

22 Since, $\angle AOM$ is 30°.



Hence, the required point B is $(-3, 3\sqrt{3})$.

23 Let (x_1, y_1), (x_2, y_2) and (x_3, y_3) are coordinates of the points D, E and F which divide each AB, BC and CA respectively in the ratio 3 : 1 (internally).



$$\therefore\ x_1 = \frac{3\times 6 - 1\times 1}{4} = \frac{17}{4}$$

$$y_1 = \frac{-2\times 3 + 4\times 1}{4} = -\frac{2}{4} = -\frac{1}{2}$$

Similarly, $x_2 = 0,\ y_2 = \dfrac{5}{2}$

and $\quad x_3 = -\dfrac{5}{4},\ y_3 = 4$

Let (x, y) be the coordinates of centroid of ΔDEF.

$$\therefore\quad x = \frac{1}{3}\left(\frac{17}{4} + 0 - \frac{5}{4}\right) = 1$$

$$\text{and } y = \frac{1}{3}\left(-\frac{1}{2} + \frac{5}{2} + 4\right) = 2$$

So, the coordinates of centroid are (1, 2).

24 The circle touching the X-axis is
$$x^2 + y^2 + 2gx + 2fy + g^2 = 0.$$
Since, it passes through (p, q).
$$\therefore\quad p^2 + q^2 + 2gp + 2fq + g^2 = 0 \quad \text{...(i)}$$
If (x, y) is the other end of the diameter, then
$$p + x = -2g,\ q + y = -2f$$
Now, Eq. (i) gives
$$p^2 + q^2 - p(p + x) - q(q + y) + \frac{(p + x)^2}{4} = 0$$
$$\Rightarrow\quad (x + p)^2 = 4px + 4qy$$
$$\Rightarrow\quad (x - p)^2 = 4qy$$

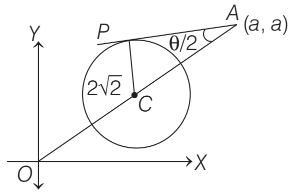
25 Given that,
$$x^2 + y^2 - 2x - 2y - 6 = 0$$
Centre = $C(1, 1)$, radius = $2\sqrt{2}$

Since, point (a, a) must lie outside the circle.

So, $\quad 2a^2 - 4a - 6 > 0$

$\Rightarrow\quad a < -1$ or $a > 3$

Now, in ΔPAC, $\tan\dfrac{\theta}{2} = \dfrac{2\sqrt{2}}{\sqrt{2a^2 - 4a - 6}}$



As given that, $\dfrac{\pi}{3} < \theta < \pi$

$$\Rightarrow\quad \frac{\pi}{6} < \frac{\theta}{2} < \frac{\pi}{2}$$

$$\therefore\quad \frac{2\sqrt{2}}{\sqrt{2a^2 - 4a - 6}} > \frac{1}{\sqrt{3}}$$

$$\Rightarrow\quad \sqrt{a^2 - 2a - 3} < 2\sqrt{3}$$

$$\therefore\quad a^2 - 2a - 3 < 12$$

$$\Rightarrow\quad a^2 - 2a - 15 < 0$$

$$\Rightarrow\quad -3 < a < 5$$

$$\therefore\quad a \in (-3, -1) \cup (3, 5)$$

26 Let the general equation of the circle be
$$x^2 + y^2 + 2gx + 2fy + c = 0.$$
$\because$ The equation of circle passing through (0, 0), (2, 0) and (0, – 2).

$\therefore\quad c = 0 \quad$ ...(i)

$4 + 4g + c = 0 \quad$ ...(ii)

and $\quad 4 - 4f + c = 0 \quad$ ...(iii)

On solving Eqs. (i), (ii) and (iii), we get $c = 0,\ g = -1,\ f = 1$

$\therefore$ The equation of circle becomes
$$x^2 + y^2 - 2x + 2y = 0$$
Since, it is passes through $(k, -2)$,
$$k^2 + 4 - 2k - 4 = 0$$
$$\Rightarrow\quad k^2 - 2k = 0 \Rightarrow k = 0, 2$$
We have already take a point $(0, -2)$, so we take only $k = 2$.

27 Slope of line $OP = \dfrac{3}{4}$, let new position is $Q(x, y)$.

Slope of $OQ = \dfrac{y}{x}$,

also $x^2 + y^2 = OQ^2 = 25 = (OP^2)$

$$\therefore\ \tan 45° = \left|\frac{\dfrac{y}{x} - \dfrac{3}{4}}{1 + \dfrac{3y}{4x}}\right|$$

$$\Rightarrow\quad \pm 1 = \frac{4y - 3x}{4x + 3y}$$

$$\Rightarrow\quad 4x + 3y = 4y - 3x$$

or $\quad -4x - 3y = 4y - 3x$

$\Rightarrow\quad x = \dfrac{1}{7}y \quad$ ...(i)

or $\quad -x = 7y \quad$ ...(ii)

Correct relation is $x = \dfrac{1}{7}y$ as new point must lies in Ist quadrant.

$$\therefore\quad x^2 + 49x^2 = 25$$

$$\Rightarrow\quad x = +\frac{1}{\sqrt{2}},\ y = \frac{7}{\sqrt{2}}$$

28 Since, $(\text{radius})^2 \le 36$

$\Rightarrow \lambda^2 + \lambda^2 - 14 \le 36$

$\Rightarrow \lambda^2 \le 25 \Rightarrow -5 \le \lambda \le 5 \Rightarrow \lambda$
$= 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5$

Hence, number of integer values of λ is 11.

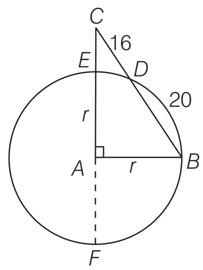
29 Tangent to the circle with slope m is
$$y = m(x - 6) \pm \sqrt{2(1 + m^2)}.$$
Since, it passes through (4, 0).
$$\therefore\quad 4m^2 = 2 + 2m^2$$
$$\Rightarrow\quad m = \pm 1$$

30 Since, $1 - |n| > 0$

$\Rightarrow\ |n| < 1$ or $n \in (-1, 1)$

31 Tangents at t and t' meet on the point (x, y) given by
$$x = att' = ak^2t'^2 \quad \text{...(i)}$$
and $\quad y = a(t + t') = a(k^2t' + t')$
$$= at'(k^2 + 1) \quad \text{...(ii)}$$
From Eqs. (i) and (ii),
$$x = \frac{ak^2 \cdot y^2}{a^2(k^2 + 1)^2} = \frac{k^2y^2}{a(k^2 + 1)^2}$$
$$\Rightarrow y^2 = \frac{ax(k^2 + 1)^2}{k^2} = ax\left(k + \frac{1}{k}\right)^2$$

32. In ΔABC
$$AC^2 + AB^2 = BC^2$$
$$AC^2 + r^2 = 36^2 \quad \text{...(i)}$$
and $\quad CF \times CE = BC \times CD$
$$\Rightarrow\quad (AC + r)(AC - r) = 36 \times 16$$
$$\Rightarrow\quad AC^2 - r^2 = 36 \times 16 \quad \text{...(ii)}$$
From Eqs. (i) and (ii), we get
$$2AC^2 = 36(36 + 16)$$
$$\Rightarrow\quad AC^2 = 18 \times 52$$
$$\Rightarrow\quad AC = 6\sqrt{26}$$



33 Let $PA = r_1,\ PB = -r_2$

Put $(\sqrt{3} + r\cos\theta, r\sin\theta)$ in $y^2 = x + 2$

$$\Rightarrow\quad r^2\sin^2\theta = (\sqrt{3} + r\cos\theta) + 2$$

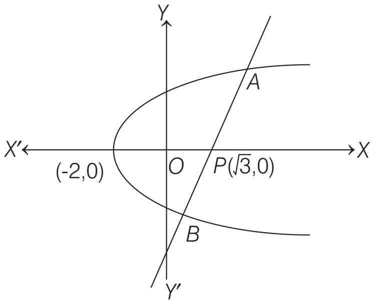
$$\Rightarrow\quad r^2\sin^2\theta - r\cos\theta - (\sqrt{3} + 2) = 0$$

$$PA \cdot PB = -r_1 . r_2 = \frac{\sqrt{3} + 2}{\sin^2\theta}$$

$$= (\sqrt{3} + 2)(1 + \cot^2\theta)$$

$= (\sqrt{3}+2)\left(1+\frac{1}{3}\right)$ $[\because \tan\theta = \sqrt{3}]$

$PA \cdot PB = \frac{4(2+\sqrt{3})}{3}$



34 $(SP)(S'P) = a(1 - e\cos\theta)\,a(1 + e\cos\theta)$

$= a^2(1 - e^2\cos^2\theta)$

$= a^2 - a^2e^2\cos^2\theta$

$= 25 - 9\cos^2\theta$

Maximum $= 25 - 9(0) = 25$ $[\theta = 90°]$

Minimum $= 25 - 9(1) = 16$ $[\theta = 0°]$

Maximum–Minimum $= 25 - 16 = 9$

35 Let the equations of two given circles are

$x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ ...(i)

and $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ ...(ii)

Now, the equations of the chords of contacts from $P(h, k)$ to Eqs. (i) and (ii) are

$x(h + g_1) + y(k + f_1) + g_1h + f_1k + c_1 = 0$

and $x(h + g_2) + y(k + f_2) + g_2h + f_2k + c_2 = 0$

According to the given condition,

$$\frac{(h + g_1)}{(k + f_1)} \times \frac{(h + g_2)}{(k + f_2)} = -1$$

$\Rightarrow h^2 + (g_1 + g_2)h + g_1g_2 + k^2 + k(f_1 + f_2) + f_1f_2 = 0$

Hence, the locus of point is

$x^2 + y^2 + (g_1 + g_2)x + (f_1 + f_2)y + g_1g_2 + f_1f_2 = 0$

which is the equation of a circle.

36 Equation of tangent at $(3\sqrt{3}\cos\theta, \sin\theta)$ to the ellipse $\frac{x^2}{27} + y^2 = 1$ is

$$\frac{x\cos\theta}{3\sqrt{3}} + y\sin\theta = 1.$$

This intersect on the coordinate axes at $(3\sqrt{3}\sec\theta, 0)$ and $(0, \operatorname{cosec}\theta)$

$\therefore$ Sum of intercepts on axes is

$3\sqrt{3}\sec\theta + \operatorname{cosec}\theta = f(\theta)$ [say]

On differentiating w.r.t. θ, we get

$f'(\theta) = 3\sqrt{3}\sec\theta\tan\theta - \operatorname{cosec}\theta\cot\theta$

$$= \frac{3\sqrt{3}\sin^3\theta - \cos^3\theta}{\sin^2\theta\cos^2\theta}$$

For maxima and minima, put $f'(\theta) = 0$

$3\sqrt{3}\sin^3\theta - \cos^3\theta = 0$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = \frac{\pi}{6}$

At $\theta = \frac{\pi}{3}$, $f''(\theta) > 0$. So, $f(\theta)$ is minimum at $\theta = \frac{\pi}{6}$.

37 Given, $x = ct$, $y = c/t$

Then, $\frac{dy}{dt} = \frac{-c}{t^2}$ and $\frac{dx}{dt} = c$

$\therefore \frac{dy}{dx} = \frac{-1}{t^2}$

But equation of tangent is $px + qy + r = 0$.

$\therefore -\frac{p}{q} = -\frac{1}{t^2} \Rightarrow \frac{p}{q} = \frac{1}{t^2} > 0$

$\Rightarrow \frac{p}{q} > 0$

$\Rightarrow p > 0, q > 0$ or $p < 0, q < 0$

38 Since, the lines $x + 3y + 2 = 0$ and $3x - y + k = 0$ are conjugate w.r.t. $\frac{x^2}{5} - \frac{y^2}{3} = 1$.

$\therefore 5(1)(3) - 3(3)(-1) = 2k$

$\Rightarrow k = 12$

Hence, equation of conjugate line is $3x - y + 12 = 0$.

39 Given, equation can be rewritten as

$\frac{x^2}{2} - \frac{y^2}{1} = 1$

Here, $a^2 = 2, b^2 = 1$

The product of length of perpendicular drawn from any point on the hyperbola to the asymptotes is

$$\frac{a^2b^2}{a^2 + b^2} = \frac{2(1)}{2+1} = \frac{2}{3}.$$

40 The equation of tangent is

$$\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1.$$

So, the coordinates of A and B are $(a\cos\theta, 0)$ and $(0, -b\cot\theta)$, respectively.

Let coordinates of P are (h, k).

$\therefore h = a\cos\theta$, $k = -b\cot\theta$

$\Rightarrow \frac{k}{h} = -\frac{b}{a\sin\theta}$

$\Rightarrow \frac{b^2h^2}{a^2k^2} = \sin^2\theta$

$\Rightarrow \frac{b^2h^2}{a^2k^2} + \frac{h^2}{a^2} = 1$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow \frac{a^2}{h^2} - \frac{b^2}{k^2} = 1$

Hence, the locus of P is $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$.

41 Image of $A(2,-1)$ with respect to line $3x - 2y + 5 = 0$. A' is given by

$$\frac{x-2}{3} = \frac{y+1}{-2} = \frac{-2(6+2+5)}{13} = -2$$

$A' \in (-4, 3)$

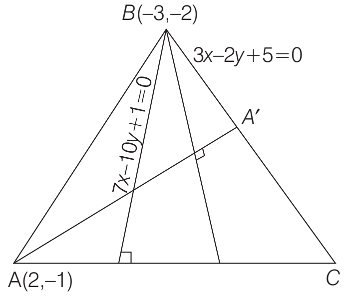
Coordinate of B is intersection point of $7x - 10y + 1 = 0$ and $3x - 2y + 5 = 0$

i.e. $(-3, -2)$

$\therefore$ Equation of BC is

$$y - 3 = \frac{3+2}{-4+3}(x+4)$$

$\Rightarrow 5x + y + 17 = 0$



42 We have, $\frac{a}{\sqrt{bc}} - 2 = \sqrt{\frac{b}{c}} + \sqrt{\frac{c}{b}}$

$\Rightarrow a - 2\sqrt{bc} = b + c$

$\Rightarrow a = b + c + 2\sqrt{bc}$

$\Rightarrow (\sqrt{a})^2 = (\sqrt{b} + \sqrt{c})^2$

$\Rightarrow (\sqrt{b} + \sqrt{c})^2 - (\sqrt{a})^2 = 0$

$\Rightarrow \sqrt{b} + \sqrt{c} - \sqrt{a} = 0$

or $\sqrt{b} + \sqrt{c} + \sqrt{a} = 0$ (rejected)

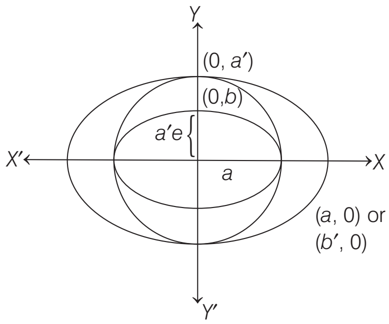
$\Rightarrow \sqrt{ax} + \sqrt{by} + \sqrt{c} = 0$

passes through fixed point $(-1, 1)$

43 Here, $b' = a$, $a'e = b$

$(b')^2 = (a')^2 - (a'e)^2$

$\Rightarrow a^2 = \frac{b^2}{e^2} - b^2$



$\Rightarrow e^2 = \frac{b^2}{a^2}(1 - e^2)$ $\left[\because 1 - e^2 = \frac{b^2}{a^2}\right]$

$\Rightarrow e^2 = (1 - e^2)(1 - e^2)$

$\Rightarrow 1 - e^2 = \pm e$

$\Rightarrow \quad e^2 - e - 1 = 0$ or $e^2 + e - 1 = 0$

$\Rightarrow \quad e = \dfrac{-1 \pm \sqrt{5}}{2}$

or $\quad \dfrac{1 \pm \sqrt{5}}{2}$

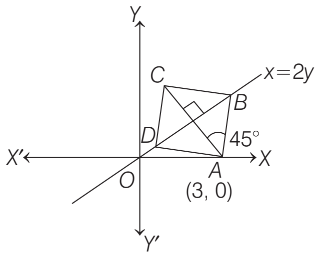
$\Rightarrow \quad e = \dfrac{\sqrt{5} - 1}{2} \qquad [0 < e < 1]$

44 Equation of diagonal

AC is $y - 0 = -2(x - 3)$

$\Rightarrow \quad 2x + y = 6$

On solving $2x + y = 6$ and $x = 2y$, we get $y = \dfrac{6}{5}$ and $x = \dfrac{12}{5}$



So, the centre of square is $\left(\dfrac{12}{5}, \dfrac{6}{5}\right)$.

Let slope of side AB or AD is m, then

$$\left|\frac{m - (-2)}{1 + m(-2)}\right| = 1$$

$\Rightarrow \quad (m + 2) = \pm (1 - 2m)$

$\Rightarrow \quad m = -\dfrac{1}{3}$ and $m = 3$

Hence, slopes of AB and AD are 3 and $-\dfrac{1}{3}$, respectively.

$\therefore$ Equations of sides AB and AD are

$y - 0 = 3(x - 3)$

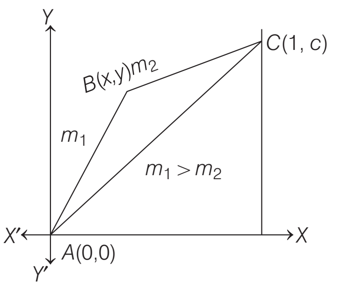
and $\quad y - 0 = -\dfrac{1}{3}(x - 3)$

or $y - 3x + 9 = 0$ and $3y + x - 3 = 0$, respectively.

45 Let the coordinate of C be (1,c)

$$m_2 = \frac{c - y}{1 - x}$$

$\Rightarrow \quad m_2 = \dfrac{c - m_1 x}{1 - x} \qquad \left(\because m_1 = \dfrac{y}{x}\right)$



$\Rightarrow \quad (m_1 - m_2)x = c - m_2$

$\Rightarrow \quad c = (m_1 - m_2)x + m_2$

Now area of

$$\Delta ABC = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ x & m_1 x & 1 \\ 1 & c & 1 \end{vmatrix} = \frac{1}{2}|cx - m_1 x|$$

$$= \frac{1}{2}|((m_1 - m_2)x + m_2)x - m_1 x|$$

$$= \frac{1}{2}(m_1 - m_2)(x - x^2)$$

$[\because x > x^2 \text{ in } (0, 1)]$

Hence $f(x) = \dfrac{1}{2}(x - x^2)$

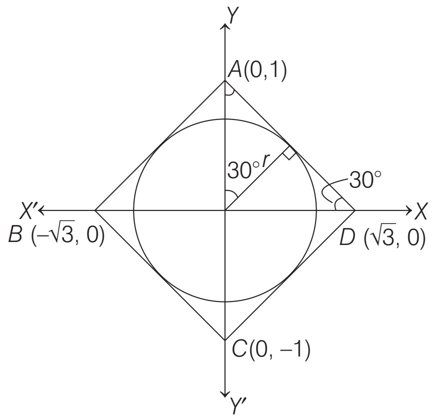
$f(x)_{\max} = \dfrac{1}{8}$,

when $x = \dfrac{1}{2}$

46 $r = \sqrt{3} \sin 30° = \dfrac{\sqrt{3}}{2}$

$P(x, y)$ is any point on circle

$(PA)^2 + (PB)^2 + (PC)^2 + (PD)^2$



$= x^2 + (y - 1)^2 + (x + \sqrt{3})^2 + y^2 + x^2 + (y + 1)^2 + (x - \sqrt{3})^2 + y^2$

$= 4(x^2 + y^2 + 2)$

$= 4(r^2 + 2) \qquad [\because x^2 + y^2 = r^2]$

$= 4\left(\dfrac{3}{4} + 2\right) = 11$

47 Any tangent to $y^2 = 4x$ is of the form

$y = mx + \dfrac{1}{m}$, $(\because a = 1)$,

This touches the circle $(x - 3)^2 + y^2 = 9$, whose centre is (3, 0) and radius is 3.

So, $\left|\dfrac{m(3) + \dfrac{1}{m} - 0}{\sqrt{m^2 + 1}}\right| = 3$

$3m^2 + 1 = \pm 3m\sqrt{m^2 + 1}$

$\Rightarrow \quad 9m^4 + 1 + 6m^2 = 9m^2(m^2 + 1)$

[on squaring both sides]

$\Rightarrow \quad 3m^2 = 1$

$\Rightarrow \quad m = \pm \dfrac{1}{\sqrt{3}}$

If the tangent touches the parabola and circle above X-axis, then slope m should be positive.

$\therefore \quad m = \dfrac{1}{\sqrt{3}}$ and the equation is

$y = \dfrac{x}{\sqrt{3}} + \sqrt{3}$

$\Rightarrow \quad \sqrt{3}y = x + 3$

which is the required equation of tangent.

48 Given equation is

$$\frac{x^2}{16} + \frac{y^2}{(16/\sqrt{11})^2} = 1.$$

Thus, the parametric coordinates are $\left(4\cos\phi, \dfrac{16}{\sqrt{11}}\sin\phi\right)$. The equation of tangent at this point is

$$\frac{x\cos\phi}{4} + \frac{\sqrt{11}y\sin\phi}{16} = 1.$$

This touches the circle

$x^2 + y^2 - 2x - 15 = 0$

$$\therefore \quad \frac{\left|\dfrac{\cos\phi}{4} - 1\right|}{\sqrt{\dfrac{\cos^2\phi}{16} + \dfrac{11\sin^2\phi}{256}}} = 4$$

$\Rightarrow \cos^2\phi + 16 - 8\cos\phi = 256\left(\dfrac{\cos^2\phi}{16} + \dfrac{11\sin^2\phi}{256}\right)$

$\Rightarrow \quad 15\cos^2\phi + 11(1 - \cos^2\phi) + 8\cos\phi - 16 = 0$

$\Rightarrow \quad 4\cos^2\phi + 8\cos\phi - 5 = 0$

$\Rightarrow \quad \cos\phi = \dfrac{1}{2} \qquad \left[\because \cos\phi \neq \dfrac{5}{2}\right]$

$\Rightarrow \quad \phi = \dfrac{\pi}{3}$ or $-\dfrac{\pi}{3}$

49 Since, $p, x_1, x_2, \dots$ and $q, y_1, y_2, \dots$ are in AP with common differences a and b, respectively.

$\Rightarrow \quad x_i = p + ai$ and $y_i = q + ib$

$\therefore \quad h = \dfrac{x_1 + x_2 + \dots + x_n}{n}$

and $\quad k = \dfrac{y_1 + y_2 + \dots + y_n}{n}$

$\Rightarrow \quad nh = \sum_{i=1}^{n} x_i$ and $nk = \sum_{i=1}^{n} y_i$

$\Rightarrow \quad nh = \sum_{i=1}^{n}(p + ia)$

and $nh = \sum_{i=1}^{n}(q + ib)$

$\Rightarrow nh = np + \frac{n(n+1)}{2}a$

and $nk = nq + \frac{n(n+1)}{2}b$

$\Rightarrow \frac{h-p}{a} = \frac{n+1}{2}$ and $\frac{k-q}{b} = \frac{n+1}{2}$

$\therefore \frac{h-p}{a} = \frac{k-q}{b}$

Hence, locus of (h, k) is

$b(x-p) = a(k-q)$.

Hence, Statement II is true and since for Statement I, $n = 3$

So, Statement I is true and Statement II is a correct explanation of Statement I.

50 If t_1 and t_2 are parameters of a and b, then

$t_1 t_2 = -4$ ...(i)

Also, $t_1\left(-t_1 - \frac{2}{t_1}\right) = -4$ ...(ii)

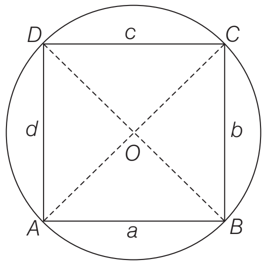
On solving Eqs. (i) and (ii), we get

$t_1 = \sqrt{2}$

Also, $m_{AB} = \frac{2}{t_2 + t_1} = -t_1 = -\sqrt{2}$

51 Clearly, $ac + bd = AC \cdot BD \leq 4$

[using Ptolemy's theorem]



$\Rightarrow ac + bd = 4$ and $AC \cdot BD = 4$

but $ac + bd \geq 4$ [$\because$ AM$\geq$ GM]

$\Rightarrow AC = BD = 2$ and $ac = bd = 2$

[$\because abcd \geq 4$]

$\Rightarrow a = b = c = d = \sqrt{2}$

52 Statement II is true.

For the point (2, 2), $t_1 = 1$

For the point (4, 1), $t_2 = 2$

For the point (6, 2/3), $t_3 = 3$

For the point $\left(\frac{1}{4}, 16\right)$, $t_4 = \frac{1}{8}$

Now, $t_1 \cdot t_2 \cdot t_3 \cdot t_4 = \frac{3}{4} \neq 1$

Hence, Statement I is false.

53 The auxiliary circle of an ellipse

$\frac{x^2}{4} + \frac{y^2}{b^2} = 1, b < 2$ is $x^2 + y^2 = 4$

54 The equation of tangent to the ellipse is

$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ and it meets the directrix $x = \frac{a}{e}$ at

$T\left[\frac{a}{e}, \frac{b(e - \cos\theta)}{e\sin\theta}\right]$.

Since, focus is $S(ae, 0)$.

$\therefore$ Slope of $SP = \frac{b\sin\theta}{a(\cos\theta - e)}$

and slope of $ST = \frac{b(e - \cos\theta)}{a\sin\theta(1 - e^2)}$

Now, as product of slopes $= -\frac{b^2}{a^2(1-e^2)}$

$= -1$, therefore PT substends a right angle at the focus.

Hence, circle with PT as diameter passes through the focus.

55 Statement II is true, using in Statement I, $(x-1)^2 + (y-4)^2$

$= (x-k)^2 + (y-3)^2$

$\Rightarrow 2(k-1)x - 2y = k^2 - 8$

y-intercept $= -\frac{k^2 - 8}{2} = -4$ [given]

$\Rightarrow k^2 = 16 \Rightarrow k^2 - 16 = 0$